MC Mechanics Answers and Explanations

1. The answer is C. Use $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(0 + 10)(5) = 25$ m

2. The answer is E. At the highest point the velocity must be 0 (or it would go higher), but the velocity is still changing. The acceleration is just due to gravity, directed down.

3. The answer is B. Work is the component of the force in the direction of the displacement, $T\cos\theta$ multiplied by the displacement, D.

4. The answer is B. Since the velocity is constant, $T\cos\theta - F_{\text{fric}} = ma = 0$. To get the friction force, you need the normal force. In the vertical direction, the forces add to 0 as well because there's no movement in this direction at all, so
   $N + T\sin\theta - mg = ma_y = 0$
   $N = mg - T\sin\theta$
   Then the friction force is $F_{\text{fric}} = \mu N = \mu(mg - T\sin\theta)$.
   Plugging into the first equation gives B.

5. The answer is D. Since the orbital speed is $v = \sqrt{\frac{GM_e}{R}}$, independent of the satellite's mass, I isn't true. Since the centripetal acceleration is proportional to v^2, which depends on the Earth's mass, II isn't true. The gravitational force exerts no torque about the Earth's center, so III is true.

6. The answer is B. At terminal speed there is 0 acceleration, so the net force is 0. Then you have $mg - bv^2 = 0 \Rightarrow v = \sqrt{\frac{mg}{b}}$.

7. The answer is D. The force is the negative slope of the graph. At $r = 2a$, the curve is well approximated by a straight line. At the $a \rightarrow 2a$ interval, the graph decreases from $3U_I \rightarrow 0$, so the slope is $-\frac{3U_1}{a}$.

8. The answer is E. Use conservation of energy, recognizing that the mass loses $4U_1$ of potential energy as it moves to the lowest PE at $r = 3a$, so
   $-\Delta U = \Delta K$
   $4U_1 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{8U_1}{m}}$

9. The answer is B. The area under the F vs. t graph is the impulse delivered by the force, which will equal the change in momentum of the object if it's the net force. The area from $3 \rightarrow 4$ s is negative, so this contribution cancels out the $0 \rightarrow 1$ s contribution. You're left with a rectangle from $1 \rightarrow 2$ s and a triangle from $2 \rightarrow 3$ s, so $10 + 5 = 15\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$.

10. The answer is D. Using momentum conservation, you have

$$P_i = P_f$$

$$6mv = m(0) + 2m(2v) + 3mv' \qquad \Rightarrow v' = \frac{2}{3}v$$

11. The answer is C. The force at the center exerts 0 torque, and the other two both exert torques directed out of the page, so they have the same sign.

$$net\tau = 2F\left(\frac{L}{4}\right) + 0 + 2F\sin 30\left(\frac{L}{4}\right) = \frac{3}{4}FL$$

12. The answer is B. Looking at the system as a whole and applying the second law gives

$$netF = Mg = (m + M)a \Rightarrow a = \frac{M}{m + M}g$$

Only the tension accelerates m, so applying the second law to it, you have

$$netF = T = ma = \frac{mM}{m + M}g$$

13. The answer is D. For circular orbit, combine the fact that $l = mvR$ and $v = \sqrt{\frac{GM_e}{R}}$.

$$l = m\sqrt{\frac{GM_e}{R}}R = m\sqrt{GM_eR}$$

14. The answer is D. From the form of the equation, $\omega^2 = 4\pi$. Since $T = \frac{2\pi}{\omega}$, D follows.

15. The answer is B. Because v is positive and constant over the first interval, x must be linearly increasing over this interval. Over the third interval, v is negative and constant, so x must be linearly decreasing. Over the second interval, the acceleration is constant and negative, which implies a parabolic shape, and the three segments must be continuous. Only B fits these categories.

16. The answer is B. The initial horizontal velocity, $v_{x0} = 50\cos 37 = 40$ m/s, remains the same throughout the motion. At the highest point, the y-component of the velocity will be 0, because otherwise it would go higher. The acceleration is provided by gravity, which acts downward with a magnitude of 10 m/s^2 throughout the motion.

17. The answer is C. Since $net\vec{F}\Delta t = \Delta\vec{p}$, you can determine the direction of the net force from the direction of $\vec{p_f} - \vec{p_i} = \vec{p_f} + (-\vec{p_i})$. C follows from the rules for vector addition.

18. The answer is D. From the form of the equation, $\omega^2 = 6\pi$. From knowledge of the harmonic oscillator equation for a spring-mass system, you have $\omega^2 = \frac{k}{m}$. Combining the two gives you D.

19. The answer is B. The second derivative of x is $a(t) = -(0.5)(6\pi)^2 \cos\left(6\pi t + \frac{\pi}{8}\right)$. The maximum value of this is $18\pi^2$ since the cosine has a maximum value of 1.

20. The answer is D. The centripetal acceleration is proportional to v^2, which begins at 0, hits a maximum after $\frac{1}{4}$ period, and returns to 0 again after $\frac{1}{2}$ period, so A and D are possible. The derivative of v^2 is obviously continuous, so A is ruled out. The cusp implies nonphysical behavior at the low point of the motion.

21. The answer is C. The tangential acceleration is just $g \sin \theta$, which has a maximum magnitude at the beginning and after $\frac{1}{2}$ period; it is 0 at the low point after $\frac{1}{4}$ period. Only C fits these categories.

22. The answer is E. The tension always acts perpendicular to the displacement, so it does no work.

23. The answer is D. $W = \int_{x_i}^{x_f} F(x)dx = \int_2^1 (2x - 8x^3)dx = (x^2 - 2x^4)_2^1 = 1 - 2 - 4 + 32 = 27 \text{ J}$

24. The answer is E. The average power is the change in KE divided by the time during which the change occurred.

$$\Delta K = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = 3\frac{1}{2}mv^2$$

Conservation of energy tells you that $\frac{1}{2}mv^2 = mgH$. Substitution gives

$$P = \frac{\Delta K}{T} = 3\frac{mgH}{T}$$

25. The answer is D. Use momentum conservation.

$$mv + 4m\left(-\frac{1}{2}v\right) = 5mv' \Rightarrow v' = -\frac{1}{5}v$$

26. The answer is A. Use a motion equation to find the angular acceleration, then the fixed axis dynamic equation to find the torque.

$$\frac{1}{2}\omega_0 = \omega_0 + \alpha \text{T} \Rightarrow \alpha = \frac{\omega_0}{2T}$$

$$net\ \tau = I\alpha = -\frac{I\omega_0}{2T}$$

27. The answer is C. $I = \sum mR^2 = mL^2 + m(2L)^2 + m(3L)^2 = 14mL^2$

28. The answer is C. The value of g at the surface of a planet is proportional to $\frac{M_p}{R_p^2}$. Taking ratios of planet-X values to those of Earth, you have

$$3 = \frac{g_X}{g_E} = \frac{\left(\frac{2M_E}{\text{R}_X^2}\right)}{\left(\frac{M_E}{R_E^2}\right)} = \frac{2R_E^2}{R_X^2}$$

29. The answer is C. If the mass is displaced an amount x from equilibrium, each spring will be stretched only $\frac{1}{2}x$. Since only one spring acts directly on the mass, it experiences a restoring force $F = -\frac{1}{2}kx$ for a displacement x, as if there were just a single spring with spring constant $\frac{1}{2}k$. The period will then be $T = 2\pi\frac{m}{\frac{1}{2}k}$.

30. The answer is D. Take two derivatives to find acceleration, and then use the second law.

$$v = \frac{dx}{dt} = 3t^2 - 8t \qquad a = \frac{dv}{dt} = 6t - 8 \qquad a(1) = -2\,\frac{\text{m}}{\text{s}^2}$$

$$netF = ma = 3(-2) = -6\text{ N}$$

31. The answer is D. In the vertical direction, the maximum height reached is proportional to the square of the initial vertical component of velocity, as you can see from the motion equation

$$v_y^2 = v_{y0}^2 - 2g\Delta y \Rightarrow \Delta y = \frac{v_{y0}^2}{2g}$$

when $v_y = 0$ at the highest point. If you call H' the maximum height when the dart is fired at 60°, you can set up the ratio

$$\frac{H'}{H} = \frac{(v_0 \sin 60)^2}{v_0^2} = \frac{3}{4}$$

32. The answer is C. As they move in tandem at 10 m/s, the CM is also moving at 10 m/s. Since the forces exerted are internal to the child-man system, the CM will continue to move at the same velocity.

33. The answer is B. Use conservation of energy and the fact that the moment of inertia about the CM of a hoop is simply MR^2 because all the mass is the same distance from the CM.

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v}{R}\right)^2 = Mv^2$$

Thus,

$$KE_{\text{trans}} = \frac{1}{2}Mv^2 = \frac{1}{2}MgH$$

34. The answer is A. The force of gravity and the spring force are both conservative, so the work they do is independent of path. Because friction was acting, the mass will rebound to a lower height than its initial height. The work done by gravity in descending to a lower height is positive. The spring ends up as it started, uncompressed, so it did 0 work.

35. The answer is C. The net torque about the geometrical center will be 0.

$$\tau_m + \tau_{\text{stick}} + \tau_M = 0$$

$$mg\left(\frac{L}{2}\right) + mg\left(\frac{L}{4}\right) - Mg\left(\frac{L}{2}\right) = 0 \quad \Rightarrow M = \frac{3}{2}m$$

MC E & M Answers and Explanations

1. The answer is C. Since $E = -\frac{dV}{dx}$ is the negative slope of the graph, the magnitude of E is greatest where the graph is steepest.

2. The answer is C. The electric field points from higher potential to lower, perpendicular to the equipotential surfaces. Electrons accelerate opposite the field direction.

3. The answer is C. The electrostatic force is conservative, so the work is independent of path. Moving from $B \rightarrow D$ takes no work because there's no potential difference.

4. The answer is E. If you know the value of $\vec{E}$ at all points, then $\oint_{\text{closed surface}} \vec{E} \cdot d\vec{A}$ can be calculated for any surface. Since this integral equals $\frac{q_{in}}{\varepsilon_0}$, you can determine the charge inside the surface.

5. The answer is D. Between the two charges, the contributions to the field from each charge reinforce each other, so they cannot cancel out. To the left of L, the two contributions oppose each other, but being closer to the larger charge ensures they will never cancel out completely. To the right of R there will be one position where the two contributions exactly cancel out.

6. The answer is B. By superposition, you can add the two individual potential contributions:

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{5Q}{\frac{d}{2}} + \frac{-3Q}{\frac{d}{2}}\right)$$

7. The answer is C. When the spheres are touched, charge will redistribute until both spheres are at the same potential. Since $V \propto \frac{1}{r}$, the charge on the smaller sphere after touching will be $\frac{1}{3}$ of what will end up on the larger sphere; the charge redistributes in a 3-to-1 ratio. Since you start with Q, this means $\frac{3}{4}Q$ on the larger sphere and $\frac{1}{4}Q$ on the smaller sphere.

8. The answer is D. The two batteries oppose each other, so the equivalent battery voltage is 20 V. The parallel pair of resistors yields $\frac{1}{R} = \frac{1}{20} + \frac{1}{5} = \frac{1}{4} \Rightarrow R = 4\ \Omega$. Since all the other resistors are in series with the 4 Ω, the equivalent resistance of the circuit is 10 Ω. Then, the current is $i = \frac{20}{10} = 2A$ in the batteries and the 2 Ω resistor. Using the power relation, you have

$$E_{2\Omega} = Pt = (i^2R_{2\Omega})t = (2^2 2)60 = 480\ \text{W}$$

9. The answer is B. This battery is recharging, so $V_T = V_B + ir = 5 + 2(0.4) = 5.8$ V.

10. The answer is A. Starting at S, you can add the voltage drops, proceeding clockwise to T. Each resistor produces a drop equal to iR, and the battery produces a drop equal to its terminal voltage. $V_{ST} = -2(2) - 5.8 - 2(3) = -15.8$ V

11. The answer is C. First, find the equivalent capacitance of the two 12 μF capacitors in series, and then combine with the 6 μF using the parallel formula.

$$\frac{1}{C_s} = \frac{1}{12} + \frac{1}{12} = \frac{1}{6} \Rightarrow 6\ \mu\text{F}$$

$$C_p = 6 + 6 = 12\ \mu\text{F}$$

12. The answer is E. The two 12 μF caps will always have equal charge on their plates because they are in series. Since the equivalent capacitance of these two is 6 μF, symmetry tells you that the top and bottom branch will also have equal charge, independent of the voltage applied.

13. The answer is D. Applying the loop law to the series circuit, you have $+V_B$ through the battery, $-iR$ across the resistor, $-\frac{Q}{C}$ across the capacitor, and $-L\frac{di}{dt}$ across the inductor. These must all add to 0.

14. The answer is C. Since the inductor keeps current from flowing through it at $t = 0$, the current in the 6 Ω and 12 Ω resistors is the same, and applying the loop law around the outer loop yields $90 - 18i = 0$.

15. The answer is D. A long time after S is closed, the inductor is behaving as a piece of zero resistance wire. At this time, the overall resistance in the circuit is 10 Ω, since the 6 Ω and the 12 Ω in parallel yield 4 Ω. This means that the battery has established 9 A in the circuit, which breaks up into a 2-to-1 ratio at the junction. The 6 Ω resistor and the inductor will have 6 A established in them. The inductor has stored energy given by

$$E = \frac{1}{2}Li^2 = \frac{1}{2}(2)(6)^2 = 36\text{ J}$$

This is the total energy consumed as the current decays to 0 when S is opened.

16. The answer is B. The field created by I_w in the region of the loop is directed out of the page using the long wire right-hand rule. Each element of the top segment will feel a force down ($\vec{dl} \times \vec{B}$), but there will always be a corresponding element on the bottom segment that will feel a force up, so these contributions cancel out. The far right segment of the loop feels a force to the left, but the left side of the loop feels a force to the right, and since it is closer to the wire, it experiences a stronger field and a stronger force, so this dominates.

17. The answer is D. Inside a wire, the field rises linearly from 0. It is continuous at the surface of the wire and then drops off as $\frac{1}{r}$ outside the wire. Notice that B drops off as $\frac{1}{r^2}$.

18. The answer is C. Using Faraday's law, you get the induced voltage. Then, use Ohm's law for the current.

$$V_{in} = \frac{d\Phi}{dt} = \frac{d}{dt}(bt^2(\pi a^2)) = 2\pi a^2 bt \Rightarrow V_{in}\left(\frac{1}{2}T\right) = \pi a^2 bt$$

$$i_{in} = \frac{V_{in}}{R} = \frac{\pi a^2 bt}{R}$$

19. The answer is B. $E(r) = -\frac{dV}{dr} = -\frac{d}{dr}\left(-Cr^{\frac{3}{2}}\right) = \frac{3}{2}Cr^{\frac{1}{2}}$

20. The answer is E. The field points radially away from the symmetry axis, so the electron will experience a force toward the axis. It will first gain speed as it moves to the axis, reaching a maximum speed at $r = 0$, and then it will slow down, stop, and move back toward the axis, eventually ending at its starting position, an oscillation.

21. The answer is D. Charges with the same $\frac{q}{m}$ ratio moving perpendicular to a magnetic field will move in circles that depend on their speeds, but the period of revolution, $T = \frac{2\pi m}{qB}$, is the same for each, independent of speed.

22. The answer is C. The field between the plates of a parallel plate capacitor is given by $E = \frac{Q}{\varepsilon_0 A}$, as you can see by applying Gauss's law to a little cylinder embedded in one of the plates. This is independent of the separation. Since $V = Ed$, as d doubles, so does V.

23. The answer is B. Use conservation of energy.

$$\Delta K = -\Delta U$$

$$\frac{1}{2}m_e v^2 = -\left(\frac{-e^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{\frac{d}{2}} - \frac{1}{d}\right) = \frac{e^2}{4\pi\varepsilon_0 d} \Rightarrow v^2 = \frac{e^2}{2\pi\varepsilon_0 m_e d}$$

24. The answer is B. Since the Q in the cavity causes charge to separate in the conductor, a closed surface surrounding Q can enclose some of the separated charge. In fact, if the surface lies inside the conductor but outside the cavity, it encloses 0 total charge. This means that I isn't true. Within the cavity, there is certainly a field, as you can see by enclosing Q in a small spherical surface. It takes work to move a test charge toward Q, implying that the potential isn't constant. Choice III is not true. Choice II is true as long as equilibrium is established, because conductors contain charges free to move.

25. The answer is A. The integration direction tells you that currents moving into the page are positive. The path then yields $\oint_{\text{path}} \vec{B} \bullet d\vec{l} = \mu_0(2 + 3 - 4) = \mu_0$.

26. The answer is A. The potential must approach $+\infty$ as you get close to $+Q$ and $-\infty$ as you get close to $-Q$. This is the only graph that does this whether you approach the charges from the left or the right.

27. The answer is D. If you imagine the capacitor as charged to Q and isolated before the dielectric is inserted, introducing the dielectric will reduce the voltage drop across the dielectric by $\frac{1}{\kappa}$, but it won't affect the other space between the plates. The total voltage drop across the plates will be $V' = \frac{V}{2} + \frac{V}{2\kappa}$, where V is the original voltage drop $\frac{Q}{C}$. From the definition of capacitance, you have

$$C' = \frac{Q}{V'} = \frac{Q}{\frac{V}{2}\left(1 + \frac{1}{\kappa}\right)} = \frac{Q}{V}\frac{2\kappa}{1 + \kappa}$$

28. The answer is B. To find the field, you need to know the potential in a region around the point, because $E = -\frac{dV}{dx}$. Just knowing a single value gives you no information about how it's changing, so I isn't possible. Choice II follows from the definition of potential difference. The work it takes to move a charge from the point to infinity is just Q times the potential difference, and since you know V at infinity is 0, the potential difference is just the negative of the potential value. A similar argument tells you that III cannot be determined because you don't know the potential at the other point.

29. The answer is D. When you apply Gauss's law to the surface, you're integrating over a cylindrical surface of radius r, so $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = E(2\pi rL)$. On the right-hand side of the equation, you must have the charge inside this surface, which is $\rho V = \rho(\pi r^2 L)$. Dividing by ε_0 gives the result.

30. The answer is D. At terminal speed, the force of gravity will equal the force on the induced current.

$$mg = i_{\text{in}}LB = \frac{BLv_T}{R}LB \Rightarrow v_T = \frac{mgR}{B^2L^2}$$

31. The answer is C. Since the voltage is fixed, you should use the power equation $P = \frac{V^2}{R}$. When the wire is made twice as long, its cross-sectional area becomes $\frac{1}{2}$ its original area, to keep the volume constant. Then,

$$R' = \rho\frac{L'}{A'} = \rho\frac{2L}{\frac{A}{2}} = 4\rho\frac{L}{A} = 4R,$$

where R is the original resistance. Thus,

$$P' = \frac{V^2}{4R} = \frac{P}{4}.$$

32. The answer is D. The electric field in the capacitor will polarize the dielectric material. This induces a surface charge density on the dielectric that is opposite in sign to the plate that surface is closest to. As a result, points within the dielectric are shielded from the original charge on the plates, reducing the field and the potential difference.

33. The answer is C. The brightness of a bulb is determined by the power i^2R it consumes from the circuit. Since all bulbs have the same resistance, more current will mean brighter when comparing bulbs. S stays the same because it is in parallel with the battery and is unaffected by the other bulbs. When V is unscrewed, the resistance in the top branch increases, so T will have less current and will get dimmer. Since U no longer shares the current established in T with V, it will get brighter.

34. The answer is C. The electric force on the electron will be in the $-y$ direction, opposite to the field, so you need the magnetic force to be in the $+y$ direction to oppose this. Since the velocity is in the $+x$ direction, to get $-\vec{v} \times \vec{B}$ in the $+y$ direction, $\vec{B}$ must be in the $+z$ direction. Notice that you can use $-\vec{v} \times \vec{B}$ because you have an electron.

35. The answer is D. As the rail moves, the charge separation induced by the magnetic field creates an electric field, and at equilibrium the electric force and magnetic force on charges in the rail will be equal: $qE = qvB \Rightarrow E = Bv$. From the magnetic force on a moving charge, you can see that negative charges initially tend to be forced to the right, so the left side becomes positively charged. This creates an electric field pointing to the right.

FR Mechanics Answers and Explanations

1. (a) Use momentum conservation before and after impact.

$$mv_0 = 2mv \qquad \Rightarrow v = \frac{v_0}{2}$$

(b) Energy is conserved after the collision, with the KE just after the collision equal to the final spring PE.

$$\frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}kX^2$$

$$k = \frac{mv_0^2}{2X^2}$$

(c) Find the difference in kinetic energy before and after the collision.

$$\Delta E = \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 - \frac{1}{2}mv_0^2 = -\frac{1}{4}mv_0^2$$

(d) With the barrier removed, there are no external horizontal forces, so the center of mass will move at a constant velocity throughout the entire process. Before the collision, the velocity of the CM is determined by

$$v_{CM} = \frac{mv_0 + 0 + 0}{6m} = \frac{1}{6}v_0$$

At maximum compression, the two masses at the ends of the spring will have 0 velocity with respect to each other; otherwise, they would continue to get closer. Since they're moving at the same velocity, this must also be the velocity of the center of mass $\frac{1}{6}v_0$.

(e) Energy is still conserved after the collision. The speed of the $2m$ mass is still $\frac{1}{2}v_0$ just after collision, and at maximum compression you now know the speed:

$$E_i = E_f$$

$$\frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}(6m)\left(\frac{v_0}{6}\right)^2 + \frac{1}{2}kX_{max}^2 = \frac{1}{2}(6m)\left(\frac{v_0}{6}\right)^2 + \frac{1}{2}\left(\frac{mv_0^2}{2X^2}\right)X_{max}^2$$

$$X_{max} = \sqrt{\frac{2}{3}}X$$

2. (a) At release, the resistive force is 0 because the velocity is 0, so the second law gives

$$netF = ma = mg \quad \Rightarrow \quad a = g$$

(b) At terminal speed, the acceleration is 0 since the velocity is no longer changing. Once again, you use the second law:

$$netF = mg - bmv_T = 0 \quad \Rightarrow v_T = \frac{g}{b}$$

(c) Now apply the second law for intermediate times.

$$netF = ma = m\frac{dv}{dt} = mg - bmv \quad \Rightarrow \frac{dv}{dt} + bv = g$$

(d) The solution to the equation is $v = \frac{g}{b}(1 - e^{-bt})$

Substitute $v = \frac{g}{2b}$ and solve for t.

$$\frac{g}{2b} = \frac{g}{b}(1 - e^{-bt}) \qquad \Rightarrow t = \frac{\ln 2}{b}$$

(e) To determine the value of b, you need to measure the terminal speed of the sphere. Assuming that terminal speed is reached fairly quickly, choose a point far enough below the surface where you can be sure that terminal speed has been reached. Measure the distance H from this point to the bottom of the tube. Release the sphere at the surface, and start the stopwatch as the sphere reaches the beginning of the H distance interval. When the sphere reaches the bottom, stop the watch and record the time T. Assuming it moved at the terminal speed over this interval, the terminal speed formula will give you b:

$$v_T = \frac{g}{b} = \frac{H}{T} \qquad \Rightarrow b = \frac{gT}{H}$$

3) (a) Use the CM formula, with the large mass as the origin.

$$X = \frac{(4m)(0) + m\left(\frac{L}{2}\right) + mL}{6m} = \frac{1}{4}L$$

(b) The moment of inertia about the CM of the system will have 3 contributions, two from the masses at each end and one from the stick. Treating the masses at the end as point masses, you have

$$I = 4m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 + I_{stick}$$

The moment of inertia of the stick about the system CM can be found from the parallel axis theorem, remembering that the moment of inertia of a uniform stick about its center is

$$I_{stick}^{center} = \frac{1}{12}mL^2$$

Thus the parallel axis theorem gives you

$$I_{stick} = \frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2 = \frac{7}{48}mL^2$$

Putting all of this together you have

$$I = 4m\left(\frac{L}{4}\right)^2 + m\left(\frac{3L}{4}\right)^2 + \frac{7}{48}mL^2 = \frac{23}{24}mL^2$$

(c) Because it is pivoted about the CM, gravity will exert no torque on it after the collision, and the angular speed acquired from the collision will remain constant. This means that the pivoted system will rotate at a constant angular velocity after the collision.

(d) There is no torque exerted about the CM, so you can use angular momentum conservation about this point. The initial angular momentum comes from the single moving mass, while the final angular momentum will have a contribution from the rebounding mass and the pivoted system as it starts to rotate.

$$L_0 = L_f$$

$$mv\left(\frac{3L}{4}\right) = m\left(\frac{-v}{3}\right)\left(\frac{3L}{4}\right) + I\omega = m\left(\frac{-v}{3}\right)\left(\frac{3L}{4}\right) + \left(\frac{23}{24}mL^2\right)\omega$$

$$\omega = \frac{24v}{23L}$$

(e) Use Newton's second law with the centripetal acceleration of the larger mass. The pin exerts a force toward the center, while the weight acts away from the center.

$$netF = 4ma = 4m\omega^2 R = 4m\left(\frac{24v}{23L}\right)^2\left(\frac{L}{4}\right)$$

$$F_{pin} - 4mg = \frac{576}{529}\frac{mv^2}{L} \qquad F_{pin} = 4mg + \frac{576}{529}\frac{mv^2}{L}$$