NAME _______________________________________ DATE _________________________ PERIOD ___________

3-3 Study Guide and Intervention

Properties of Logarithms

Properties of Logarithms Since logarithms and exponents have an inverse relationship, they have certain properties that can be used to make them easier to simplify and solve.

If b, x, and y are positive real numbers, $b \neq 1$, and p is a real number, then the following statements are true.

- $\log_b xy = \log_b x + \log_b y$ — Product Property
- $\log_b \frac{x}{y} = \log_b x - \log_b y$ — Quotient Property
- $\log_b x^p = p \log_b x$ — Power Property

Example 1: Evaluate $3 \log_2 8 + 5 \log_2 \frac{1}{2}$.

$3 \log_2 8 + 5 \log_2 \frac{1}{2} = 3 \log_2 2^3 + 5 \log_2 2^{-1}$	$8 = 2^3$; $2^{-1} = \frac{1}{2}$
$= 3(3 \log_2 2) + 5(-\log_2 2)$	Power Property
$= 3(3)(1) + 5(-1)(1)$	$\log_x x = 1$
$= 4$	Simplify.

Example 2: Expand $\ln \frac{8x^5}{3y^2}$.

$\ln \frac{8x^5}{3y^2} = \ln 8x^5 - \ln 3y^2$	Quotient Property
$= \ln 8 + \ln x^5 - \ln 3 - \ln y^2$	Product Property
$= \ln 8 + 5 \ln x - \ln 3 - 2 \ln y$	Power Property

Exercises

1. Evaluate $2 \log_3 27 + 4 \log_3 \frac{1}{3}$.

Expand each expression.

2. $\log_3 \frac{5r^5}{\sqrt[3]{t^2}}$

$\log_3 5 + 5 \log_3 r - \frac{2}{3} \log_3 t$

3. $\log \frac{(a-2)(b+4)^6}{9(b-2)^5}$

$\log (a - 2) + 6 \log (b + 4) - \log 9 - 5 \log (b - 2)$

Condense each expression.

4. $11 \log_9 (x - 3) - 5 \log_9 2x$

$\log_9 \frac{(x-3)^{11}}{32x^5}$

5. $\frac{3}{4} \ln (2h - k) + \frac{3}{5} \ln (2h + k)$

$\ln \left(\sqrt[4]{(2h-k)^3}\ \sqrt[5]{(2h+k)^3}\right)$

3-3 Study Guide and Intervention *(continued)*

Properties of Logarithms

Change of Base Formula If the logarithm is in a base that needs to be changed to a different base, the **Change of Base Formula** is required.

For any positive real numbers a, b, and x, $a \neq 1$, $b \neq 1$, $\log_b x = \frac{\log_a x}{\log_a b}$.

Many non-graphing calculators cannot be used for logarithms that are not base e or base 10. Therefore, you will often use this formula, especially for scientific applications. Either of the following forms will provide the correct answer.

$$\log_b x = \frac{\log x}{\log b} \qquad\qquad \log_b x = \frac{\ln x}{\ln b}$$

Example: Evaluate each logarithm.

a. $\log_2 7$

$\log_2 7 = \frac{\ln 7}{\ln 2}$ Change of Base Formula

≈ 2.81 Use a calculator.

b. $\log_{\frac{1}{3}} 10$

$\log_{\frac{1}{3}} 10 = \frac{\log 10}{\log \frac{1}{3}}$ Change of Base Formula

≈ -2.10 Use a calculator.

Exercises

Evaluate each logarithm.

1. $\log_{32} 631$ 1.86

2. $\log_3 17$ 2.58

3. $\log_7 1094$ 3.60

4. $\log_6 94$ 2.54

5. $\log_5 256$ 3.45

6. $\log_9 712$ 2.99

7. $\log_6 832$ 3.753

8. $\log_{11} 47$ 1.606

9. $\log_3 9$ 2

10. $\log_8 256$ 2.667

11. $\log_{12} 4302$ 3.367

12. $\log_{0.5} 420$ –8.714